Electronics Cooling Cheat Sheet[1]

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Introduction

Heat conduction equation / Fourier's law
(A: area, T: temperature, $[\dot{Q}] = \mathrm{W}$):

$$\dot{Q}_{\mathrm{cond}} = -kA\frac{\mathrm{d}T}{\mathrm{d}x}$$

Thermal conductivity k:

$$[k] = \frac{\mathrm{W}}{\mathrm{m \cdot K}}$$

Convection heat flow (A_{s}: surface area, T_{s}: surface temperature, T_{amb}: ambient):

$$\dot{Q}_{\mathrm{conv}} = hA_{\mathrm{s}} \cdot (T_{\mathrm{s}} - T_{\mathrm{amb}})$$

Heat transfer coefficient h:

$$[h] = \frac{\mathrm{W}}{\mathrm{m^2 \cdot K}}$$

Forced convection over flat plate (U: air speed, $[U] = \frac{\mathrm{m}}{\mathrm{s}}$,
L: lenght of plate, $[L] = \mathrm{m}$):

- Laminar flow: $h = 3.9 \cdot \sqrt{\frac{U}{L}}$
- Turbulent flow: $h = 5.5 \cdot \left(\frac{U^4}{L}\right)^{1/5}$

Natural convection from vertical flat plate (L: height of plate, $[L] = \mathrm{m}$, T_{s}: surface temperature, T_{amb}: ambient temperature):

- Laminar flow: $h = 1.4 \cdot \left(\frac{T_{\mathrm{s}} - T_{\mathrm{amb}}}{L}\right)^{1/4}$
- Turbulent flow: $h = 1.1 \cdot (T_{\mathrm{s}} - T_{\mathrm{amb}})^{1/3}$

Radiation heat transfer (ε: surface emissivity, $[\varepsilon] = 1$,
$\sigma = 5.67 \cdot 10^{-8}\,\frac{\mathrm{W}}{\mathrm{m^2 K^4}}$: Stefan-Boltzmann constant,
A_{s}: surface area, T_{s}: surface temperature,
T_{surr}: surrounding temperature, $[T_{\mathrm{s}}] = [T_{\mathrm{surr}}] = \mathrm{K}$):

$$\dot{Q}_{\mathrm{rad}} = \varepsilon\sigma A_{\mathrm{s}} \cdot \left(T_{\mathrm{s}}^4 - T_{\mathrm{surr}}^4\right) = h_{\mathrm{rad}}\,A_{\mathrm{s}} \cdot (T_{\mathrm{s}} - T_{\mathrm{surr}})$$

Heat losses

Joule's law:

$$P = V \cdot I = R \cdot I^2 = \frac{V^2}{R}$$

Average IGBT conduction loss:

$$P_{\mathrm{avg}} = u_{\mathrm{ce},0} \cdot I_{\mathrm{c,avg}} + r_{\mathrm{c}} \cdot I_{\mathrm{c,rms}}^2$$

Average diode conduction loss:

$$P_{\mathrm{avg}} = u_{\mathrm{D},0} \cdot I_{\mathrm{D,avg}} + r_{\mathrm{D}} \cdot I_{\mathrm{D,rms}}^2$$

IGBT switching loss:

- $E_{\mathrm{on}} = \sum_{i=1}^{n} k_{\mathrm{on}} \cdot i_{\mathrm{IGBT}}(t_i)$
- $E_{\mathrm{off}} = \sum_{i=1}^{n} k_{\mathrm{off}} \cdot i_{\mathrm{IGBT}}(t_i)$
- $P_{\mathrm{sw}} = \frac{1}{T}(E_{\mathrm{on}} + E_{\mathrm{off}})$

Diode switching loss:

- $E_{\mathrm{rr}} = \sum_{i=1}^{n} k_{\mathrm{rr}} \cdot i_{\mathrm{DIode}}(t_i)$
- $P_{\mathrm{rr}} = \frac{E_{\mathrm{rr}}}{T}$

Thermal resistance networks

Thermal resistance R_{th}:

- $\dot{Q} = \frac{1}{R_{\mathrm{th}}} \cdot \Delta T$
- $[R_{\mathrm{th}}] = \frac{\mathrm{K}}{\mathrm{W}}$

Thermal resistivity r_{th} ($[\dot{q}] = \frac{\mathrm{W}}{\mathrm{m^2}}$):

- $\dot{q} = \frac{1}{r_{\mathrm{th}}} \cdot \Delta T$
- $[r_{\mathrm{th}}] = \frac{\mathrm{K \cdot m^2}}{\mathrm{W}}$

Thermal resistance of conduction (t: thickness, k: thermal coductivity, A: area):

$$R_{\mathrm{th,cond}} = \frac{t}{kA} = \frac{r_{\mathrm{th}}}{A}$$

Thermal resistivity of conduction (t: thickness, k: thermal conductivity):

$$r_{\mathrm{th,cond}} = \frac{t}{k}$$

Thermal resistance of convection (h: heat transfer coefficient, A_{s}: surface area):

$$R_{\mathrm{th,conv}} = \frac{1}{hA_{\mathrm{s}}}$$

Conversion of Celcius temperatures to Kelvin:

$$\frac{T}{\mathrm{K}} = \frac{\vartheta}{^\circ\mathrm{C}} + 273.15$$

Radiation heat transfer coefficient ($[T_{\mathrm{s}}] = [T_{\mathrm{surr}}] = \mathrm{K}$):

$$h_{\mathrm{rad}} = \varepsilon\sigma(T_{\mathrm{s}}^2 + T_{\mathrm{surr}}^2)(T_{\mathrm{s}} + T_{\mathrm{surr}})$$

Thermal resistance of radiation:

$$R_{\mathrm{th,rad}} = \frac{1}{h_{\mathrm{rad}}A}$$

Modified heat transfer coefficient ($[T_{\mathrm{s}}] = [T_{\mathrm{surr}}] = \mathrm{K}$):

$$h'_{\mathrm{rad}} = \frac{\varepsilon\sigma(T_{\mathrm{s}}^4 - T_{\mathrm{surr}}^4)}{T_{\mathrm{s}} - T_{\mathrm{amb}}}$$

Modified conduction thermal resistance:

$$R'_{\mathrm{th,rad}} = \frac{1}{h'_{\mathrm{rad}} \cdot A}$$

N thermal resistances in series:

$$R_{\mathrm{th,tot}} = \sum_{i=1}^{N} R_{\mathrm{th},i}$$

N thermal resistances in parallel:

$$\frac{1}{R_{\mathrm{th,tot}}} = \sum_{i=1}^{N} \frac{1}{R_{\mathrm{th},i}}$$

Effective normal thermal conductivity of a PCB (t: total thickness, t_{e}: total thickness of epoxy layers, t_{c}: total thickness of copper layers, k_{e}: thermal conductivity of epoxy, k_{c}: thermal conductivity of copper):

$$k_{\mathrm{n}} = \frac{t}{\frac{t_{\mathrm{e}}}{k_{\mathrm{e}}} + \frac{t_{\mathrm{c}}}{k_{\mathrm{c}}}}$$

Effective planar thermal conductivity of a PCB:

$$k_{\mathrm{p}} = \frac{k_{\mathrm{e}}t_{\mathrm{e}} + k_{\mathrm{c}}t_{\mathrm{c}}}{t}$$

Heat spreading:

- A_1: heat source area, A_2: heat spreader area
- k: thermal conductivity of heat spreader material
- t: heat spreader thickness
- h_{eff}: effective heat transfer coefficient
- $r_1 = \sqrt{\frac{A_1}{\pi}}$, $r_2 = \sqrt{\frac{A_2}{\pi}}$
- $\varepsilon = \frac{r_1}{r_2}$
- $\tau = \frac{t}{r_2}$
- $Bi = \frac{h_{\mathrm{eff}} \cdot r_2}{k}$
- $\lambda = \pi + \frac{1}{\varepsilon\sqrt{\pi}}$
- $\phi = \frac{\tanh(\lambda\tau) + \frac{\lambda}{Bi}}{1 + \frac{\lambda}{Bi}\tanh(\lambda\tau)}$
- $R_{\mathrm{th,sp}} = \frac{(1-\varepsilon)\phi}{\pi k r_1}$

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[1] `https://github.com/m-thu/sandbox/blob/master/electronics-cooling.tex`

Heat Sinks

Heat sink parameters:

- A_c: cross sectional area of one fin ($[A_\mathrm{c}] = \mathrm{m}^2$)
- P: perimeter of fin cross-section ($[P] = \mathrm{m}$)
- c: fin length
- k: thermal conductivity of heatsink material
- A_b: base plate area
- n_f: number of fins

Thermal resistance of heat sink (with *adiabatic approximation*):

- Corrected fin length: $c_\mathrm{c} = c + \frac{A_\mathrm{c}}{P}$
- $a = \sqrt{\frac{hP}{kA_\mathrm{c}}}$, $[a] = \frac{1}{\mathrm{m}}$
- Fin efficiency: $\eta_\mathrm{f} = \frac{\tanh(a \cdot c_\mathrm{c})}{a \cdot c_\mathrm{c}}$
- Fin surface area: $A_\mathrm{f} = P \cdot c_\mathrm{c}$
- Total heat sink area: $A_\mathrm{tot} = n_\mathrm{f} A_\mathrm{f} + A_\mathrm{b} - n_\mathrm{f} A_\mathrm{c}$
- Heat sink efficiency: $\eta_\mathrm{hs} = 1 - \frac{n_\mathrm{f} A_\mathrm{f}}{A_\mathrm{tot}}(1 - \eta_\mathrm{f})$
- $\Rightarrow$ $R_\mathrm{hs,conv} = \frac{1}{\eta_\mathrm{hs} A_\mathrm{tot} h}$

Caloric thermal resistance (c_p: specific heat of fluid, $[c_\mathrm{p}] = \frac{\mathrm{J}}{\mathrm{kg \cdot K}}$, $\dot{V}$: volume flow rate, $[\dot{V}] = \frac{\mathrm{m}^3}{\mathrm{s}}$, ϱ: density, $[\varrho] = \frac{\mathrm{kg}}{\mathrm{m}^3}$):

- Flow cross section: A_flow
- Air speed: U_air, $[U] = \frac{\mathrm{m}}{\mathrm{s}}$
- Volume flow rate: $\dot{V} = U_\mathrm{air} \cdot A_\mathrm{flow}$
- Mass flow rate: $\dot{m} = \dot{V} \cdot \varrho$
- $\Rightarrow$ $R_\mathrm{hs,cal} = \frac{1}{2\dot{m}c_\mathrm{p}} = \frac{1}{2\dot{V}\varrho c_\mathrm{p}}$

Effective heat transfer coefficient (for spreading resistance):

$$h_\mathrm{eff} = \frac{1}{(R_\mathrm{hs,conv} + R_\mathrm{hs,cal}) \cdot A_\mathrm{b}}$$

Linear fit for fan curve (p_max: maximum pressure when fan is completely blocked, $[p_\mathrm{max}] = \mathrm{Pa}$, $\dot{V}_\mathrm{max}$: maximum volume flow rate when fan is unobstructed, $[\dot{V}_\mathrm{max}] = \frac{\mathrm{m}^3}{\mathrm{s}}$):

$$\Delta p_\mathrm{fan}\left(\dot{V}\right) = p_\mathrm{max} - \frac{p_\mathrm{max}}{\dot{V}_\mathrm{max}} \cdot \dot{V}$$

Heat sink parameters:

- c: fin length
- s: fin spacing
- n_f: number of fins
- $n = n_\mathrm{f} - 1$: number of channels
- L: length of heatsink (in flow direction)

Hydraulic diameter:

$$d_\mathrm{h} = \frac{2sc}{s+c}$$

Fan pressure drop with laminar flow (ϱ: air density):

$$\Delta p_\mathrm{lam}\left(\dot{V}\right) = 1.5\,\frac{32\varrho\nu L}{n(sc)d_\mathrm{h}^2} \cdot \dot{V}$$

Fan operating point (laminar flow):

$$\Delta p_\mathrm{fan}\left(\dot{V}\right) = \Delta p_\mathrm{lam} \;\curvearrowright\; \dot{V} \;\curvearrowright\; p$$

Reynolds number (ν: kinematic viscosity, $[\nu] = \frac{\mathrm{m}^2}{\mathrm{s}}$):

$$Re = \frac{2\dot{V}}{n(s+c)\nu}$$

Flow type:

- $Re < 2300$: laminar flow
- $Re > 2300$: turbulent flow

Nusselt number (Pr: Prandtl's number, $[Pr] = 1$, $[Nu] = 1$):

- $X = \frac{L}{d_\mathrm{h} Re_\mathrm{lam} Pr}$
- $Nu_\mathrm{lam} = \frac{3.657\left[\tanh\left(2.264 \cdot X^{1/3} + 1.7 \cdot X^{2/3}\right)\right]^{-1} + \frac{0.0499}{X}\tanh(X)}{\tanh\left[2.432 \cdot Pr^{1/6} \cdot X^{1/6}\right]}$

Heat transfer coefficient of the heatsink (k_air: thermal conductivity of air):

$$h = \frac{Nu_\mathrm{lam} \cdot k_\mathrm{air}}{d_\mathrm{h}}$$